Thermodynamics of an Ideal Gas

Ideal Gas

The hypothetical condition approached by real gases at high temperatures and low pressures (PV = nRT)

The average translational energy for one mole of gas at a given temperature in Kelvins

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Molar Heat Capacity (C)

The energy required to raise the temperature of 1mole of a substance by 1K.

$$q = n\,C\,\Delta T$$

heat — q

moles — n

change in temperature — ΔT

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J/mol K

Heating an Ideal Gas at constant volume

$$(KE)_{ave} = \frac{3}{2} RT$$

$$\text{(heat energy required)} = \frac{3}{2} RT$$

No change in volume - no work done

$$C_v = \frac{3}{2} R$$

heat energy required to change the temp. of one mole of monoatomic gas 1 K at constant volume

Heating an Ideal Gas at constant pressure

the volume increases work is done

(heat energy required) =

(heat energy required to change the translation energy) + (the energy needed to do the PV work)

Heating an Ideal Gas at constant pressure

the volume increases work is done

(heat energy required) $= \frac{3}{2} R + P\Delta V$

$P\Delta V = nR\Delta T$

$= R$

For a 1 mole 1 K change

$C_p = \frac{3}{2} R + R$

$C_p = C_v + R$

heat energy required to change the temp. of one mole of monoatomic gas 1 K at constant pressure

Heating a Polyatomic Gas

polyatomic gases absorb energy to excite rotational and vibrational motions in addition to translational motions **causing higher C_v than (3/2)R**

$$C_p = C_v + R$$

Assuming ideal behavior if C_v is known C_p can be calculated for any gas

Heating a Gas and Energy

$$\Delta E = \frac{3}{2} R \Delta T$$

for *n* moles

$$\Delta E = C_v \Delta T \qquad \Delta E = nC_v \Delta T$$

At constant pressure work is done

(heat energy required) $= q_p = nC_p \Delta T$

$$= n(C_v + R)\,\Delta T$$

$$\Delta H = \underbrace{nC_v \Delta T}_{\Delta E} + \underbrace{nR\,\Delta T}_{P\Delta V}$$

Heating a Gas and Enthalpy

$$H = E + PV$$

$$\Delta H = \Delta E + \Delta (PV)$$

$$\Delta H = \Delta E + nR\Delta T$$

$$\Delta H = nC_v\Delta T + nR\Delta T$$

$$\Delta H = n(C_v + R)\Delta T$$

$$\Delta H = nC_p\Delta T$$

Summary

$$q = nC\Delta T$$

$$\Delta E = nC_v\Delta T$$

$$\Delta H = nC_p\Delta T$$

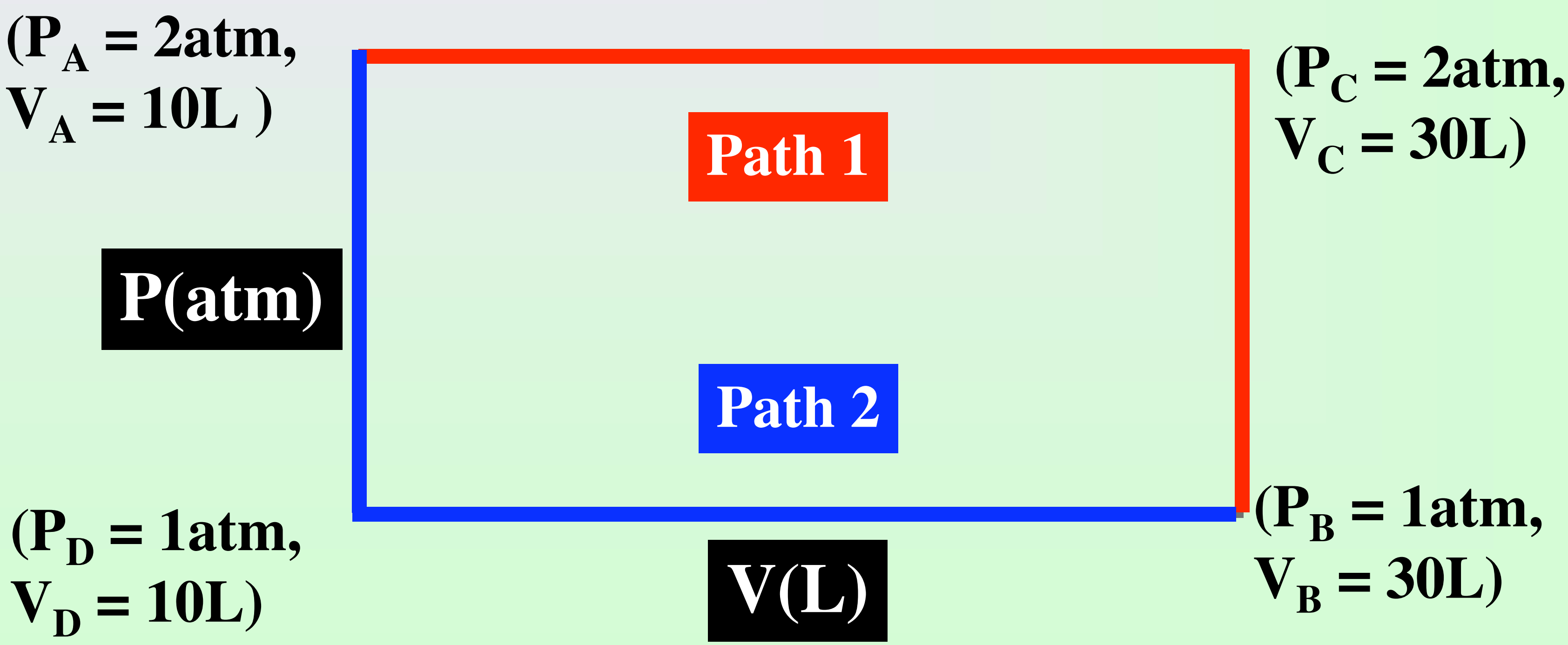
$$C_v = \frac{3}{2}R$$

$$C_p = C_v + R$$

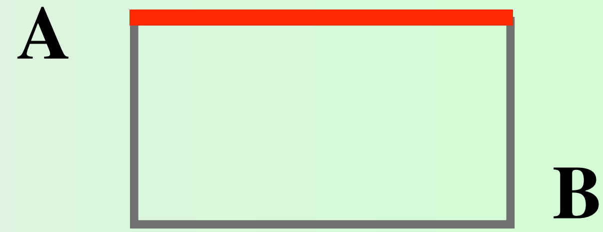
Example: Heating an Ideal Gas

Consider 2.00 mol of a monoatomic ideal gas that is taken from state A (P_A = 2atm, V_A = 10L)to state B (P_B = 1atm, V_B = 30L) by two different pathways:

calculate q, w, ΔE, and ΔH for the two pathways



Step 1



$V_A = 10L \quad V_C = 30L$

At constant pressure = 2atm

$$P\Delta V = nR\Delta T$$

$$P\Delta V = (2atm)(20L) = 40 \text{ atm L} \times \frac{101.3 \text{ J}}{1\text{atmL}} = 4.05 \times 10^3 \text{ J}$$

$$nR\Delta T = 4.05 \times 10^3 \text{ J}$$

$$\Delta T = \frac{4.05 \times 10^3 \text{ J}}{nR}$$

$$w = -P\Delta V = -4.05 \times 10^3 \text{ J}$$

$$\Delta H = q_p = 1.01 \times 10^4 \text{ J}$$

$$q_p = nC_p\Delta T = n\left(\frac{5}{2}R\right)\left(\frac{4.05 \times 10^3 \text{ J}}{nR}\right) = 1.01 \times 10^4 \text{ J}$$

$$\Delta E = nC_v\Delta T = n\left(\frac{3}{2}R\right)\left(\frac{4.05 \times 10^3 \text{ J}}{nR}\right) = 6.08 \times 10^3 \text{ J}$$

Step 2

$P_C = 2\text{atm} \quad P_B = 1\text{atm}$

At constant volume = 30L

$$\Delta PV = nR\Delta T \qquad \Delta T = \frac{\Delta PV}{nR}$$

$$\Delta T = \frac{(30\text{L})(1\text{atm} - 2\text{atm})}{nR} = \frac{-30 \text{ atmL}}{nR} \times \frac{101.3 \text{ J}}{1\text{atmL}} = \frac{-3.04 \times 10^3 \text{J}}{nR}$$

$w = 0$ (no change in volume)

$$q_v = nC_v\Delta T = n\left(\frac{3}{2}R\right)\left(\frac{-3.04 \times 10^3 \text{ J}}{nR}\right) = -4.56 \times 10^3 \text{ J}$$

$$\Delta E = q_v$$

$$\Delta H = nC_p\Delta T = n\left(\frac{5}{2}R\right)\left(\frac{-3.04 \times 10^3 \text{ J}}{nR}\right) = -7.6 \times 10^3 \text{J}$$

Step 3

$P_C = 2atm \quad P_B = 1atm$

At constant volume = 10L

$$\Delta PV = nR\Delta T \qquad \Delta T = \frac{\Delta PV}{nR}$$

$$\Delta T = \frac{(10L)(1atm - 2atm)}{nR} = \frac{-10\ atmL}{nR} \times \frac{101.3\ J}{1atmL} = \frac{-1.01 \times 10^3 J}{nR}$$

w = 0 (no change in volume)

$$q_v = nC_v\Delta T = n\left(\frac{3}{2}R\right)\left(\frac{-1.01 \times 10^3\ J}{nR}\right) = -1.52 \times 10^3\ J$$

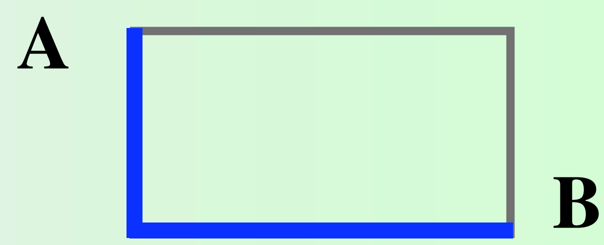
$$\Delta E = q_v$$

$$\Delta H = nC_p\Delta T = n\left(\frac{5}{2}R\right)\left(\frac{-1.01 \times 10^3\ J}{nR}\right) = -5.08 \times 10^3 J$$

Step 4

$V_A = 10L \quad V_C = 30L$

At constant pressure = 1atm



$$P\Delta V = nR\Delta T$$

$$P\Delta V = (1atm)(20L) = 20\ atm\ L \times \frac{101.3\ J}{1atmL} = 2.03 \times 10^3\ J$$

$$nR\Delta T = 2.03 \times 10^3\ J$$

$$\Delta T = \frac{2.03 \times 10^3\ J}{nR}$$

$$w = -P\Delta V = -2.03 \times 10^3 J$$

$$\Delta H = q_p = 5.08 \times 10^3\ J$$

$$q_p = nC_p\Delta T = n\left(\frac{5}{2}R\right)\left(\frac{2.03 \times 10^3\ J}{nR}\right) = 5.08 \times 10^3\ J$$

$$\Delta E = nC_v\Delta T = n\left(\frac{3}{2}R\right)\left(\frac{2.03 \times 10^3\ J}{nR}\right) = 6.08 \times 10^3\ J$$

Pathway One

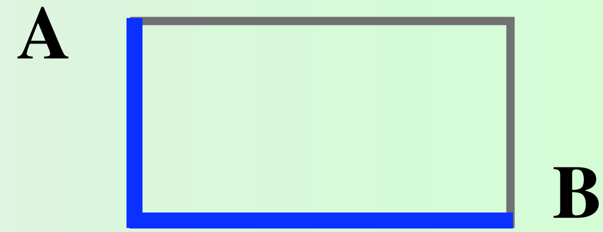
$$w = w_1 + w_2 = -4.05 \times 10^3\ J + 0$$

$$q = q_1 + q_2 = -4.56 \times 10^3\ J + 1.01 \times 10^4\ J = 5.5 \times 10^3 J$$

$$\Delta E = q + w = 5.50 \times 10^3 J - 4.05 \times 10^3\ J = 1.50 \times 10^3 J$$

$$\Delta H = \Delta H_1 + \Delta H_2 = 1.01 \times 10^4\ J + 7.6 \times 10^3 J = 2.50 \times 10^3\ J$$

Pathway Two



$$w = w_3 + w_4 = 0 - 2.03 \times 10^3 \text{ J}$$

$$q = q_3 + q_4 = -1.52 \times 10^3 \text{ J} + 5.08 \times 10^4 \text{ J} = 3.56 \times 10^3 \text{J}$$

$$\Delta E = q + w = 3.56 \times 10^3 \text{J} - 2.03 \times 10^3 \text{ J} = 1.5 \times 10^3 \text{J}$$

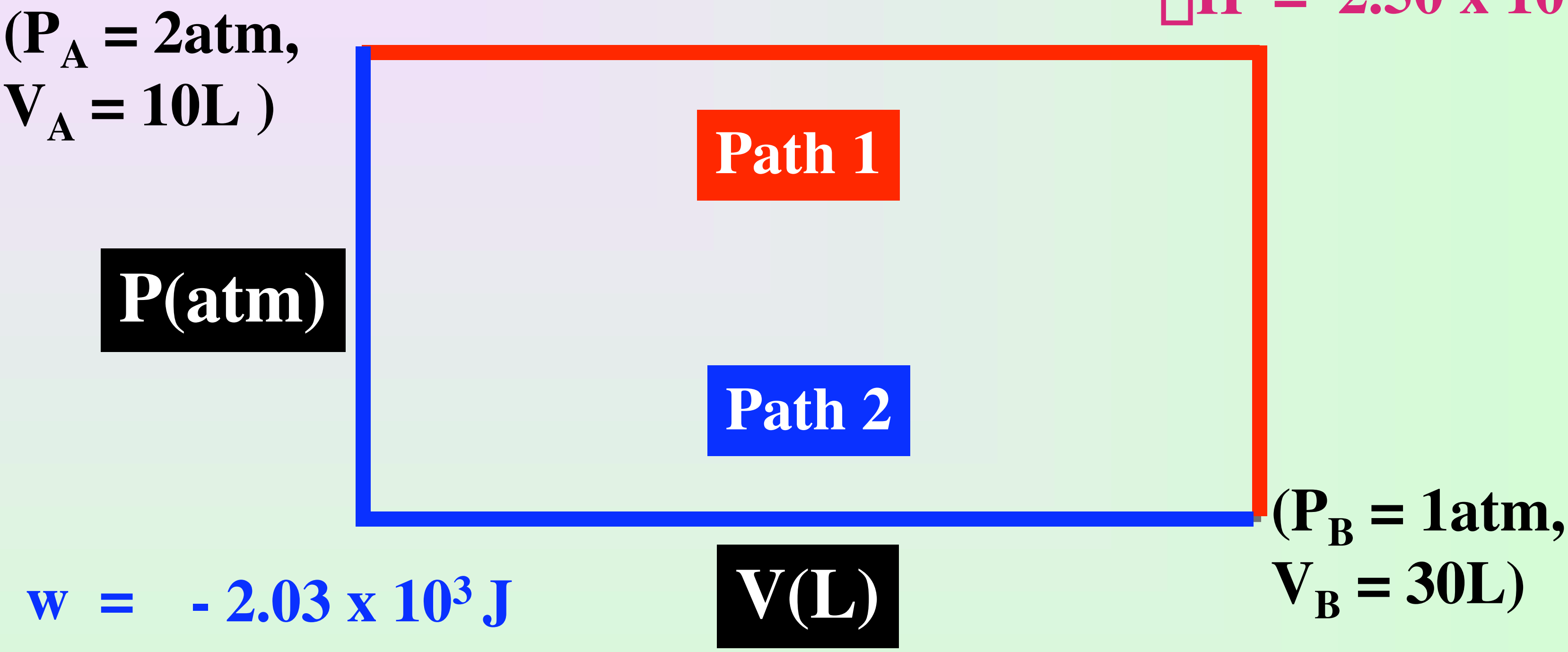
$$\Delta H = \Delta H_3 + \Delta H_4 = 2.53 \times 10^3 \text{ J} + 5.08 \times 10^3 \text{J} = 2.55 \times 10^3 \text{ J}$$

$w = -4.05 \times 10^3$ J

$q = 5.5 \times 10^3$ J

$\Delta E = 1.50 \times 10^3$ J

$\Delta H = 2.50 \times 10^3$ J



$w = - 2.03 \times 10^3$ J

$q = 3.56 \times 10^3$ J

$\Delta E = 1.5 \times 10^3$ J

$\Delta H = 2.55 \times 10^3$ J

A State Function

Does not depend on how the system arrived at its present state; only on the characteristics of the present state.

Volume, Pressure, Temperature, ΔE, ΔH